

Solutions to Problem Set 10

ECON 337901 - Financial Economics
Boston College, Department of Economics

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Due Tuesday, April 9

1. Risk Aversion and Portfolio Allocation, Part I

The investor chooses a , the amount allocated to stocks, to maximize expected utility

$$E\{u[Y_0(1+r_f) + a(\tilde{r} - r_f)]\} = (1/2) \ln[110 + a(0.30 - 0.10)] + (1/2) \ln[110 + a(0.05 - 0.10)].$$

The first-order condition

$$\left(\frac{1}{2}\right) \left(\frac{0.20}{110 + 0.20a^*}\right) - \left(\frac{1}{2}\right) \left(\frac{0.05}{110 - 0.05a^*}\right) = 0$$

leads to the solution via

$$0.20(110 - 0.05a^*) = 0.05(110 + 0.20a^*)$$

$$4(110 - 0.05a^*) = 110 + 0.20a^*$$

$$440 - 0.20a^* = 110 + 0.20a^*$$

$$0.40a^* = 330$$

$$a^* = 825.$$

2. Risk Aversion and Portfolio Allocation, Part II

Now the investor chooses a to maximize

$$\left(\frac{1}{2}\right) \left(-\frac{1}{110 + 0.20a} + 1\right) + \left(\frac{1}{2}\right) \left(-\frac{1}{110 - 0.05a} + 1\right).$$

The first-order condition

$$\left(\frac{1}{2}\right) \left[\frac{0.20}{(110 + 0.20a^*)^2}\right] - \left(\frac{1}{2}\right) \left[\frac{0.05}{(110 - 0.05a^*)^2}\right] = 0$$

leads to the solution via

$$0.20(110 - 0.05a^*)^2 = 0.05(110 + 0.20a^*)^2$$

$$4(110 - 0.05a^*)^2 = (110 + 0.20a^*)^2$$

$$2(110 - 0.05a^*) = 110 + 0.20a^*$$

$$220 - 0.10a^* = 110 + 0.20a^*$$

$$0.30a^* = 110$$

$$a^* = 366.67.$$

The utility functions in these first two questions are both of the CRRA form, but the logarithmic function from question 1 has coefficient of relative risk aversion equal to one and the utility function from question 2 has coefficient of relative risk aversion equal to two. Since the investor in question 2 is more risk averse, he or she allocates fewer dollars to risky assets.