Solutions to Problem Set 10

ECON 337901 - Financial Economics | Peter Ireland
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For Extra Practice - Not Collected or Graded

1. Interpreting Measures of Risk Aversion

With vN-M expected utility and Bernoulli utility function

$$u(Y) = \frac{Y^{1-\gamma} - 1}{1-\gamma}$$

and initial wealth $Y_0 = 10$, expected utility from accepting the lottery $(0.1, -0.1, \pi^*)$ is

$$\pi^* \left(\frac{10.1^{1-\gamma} - 1}{1-\gamma} \right) + (1-\pi^*) \left(\frac{9.9^{1-\gamma} - 1}{1-\gamma} \right)$$

while expected utility from rejecting the lottery is just

$$\frac{10^{1-\gamma} - 1}{1-\gamma}.$$

Hence, if the investor is indifferent between accepting and rejecting

$$\pi^* \left(\frac{10.1^{1-\gamma} - 1}{1-\gamma} \right) + (1-\pi^*) \left(\frac{9.9^{1-\gamma} - 1}{1-\gamma} \right) = \frac{10^{1-\gamma} - 1}{1-\gamma}$$

or, more simply,

$$\pi^*(10.1^{1-\gamma}) + (1-\pi^*)(9.9^{1-\gamma}) = 10^{1-\gamma}.$$

Rearranging this last expression yields an exact formula for π^*:

$$\pi^* = \frac{10^{1-\gamma} - 9.9^{1-\gamma}}{10.1^{1-\gamma} - 9.9^{1-\gamma}},$$

and for any setting for γ, this exact value can be compared to the value implied by the approximation

$$\pi^* \approx \frac{1}{2} + \frac{\gamma k}{4}$$

where $k = 0.01$. Results from the calculations confirm that more risk averse investors require a higher value of π^* and also confirm the accuracy of the approximation:

γ	Exact π^*	Approximate π^*
1/2	0.5013	0.5012
2	0.5050	0.5050
3	0.5075	0.5075
10	0.5250	0.5250
20	0.5498	0.5500

2. Insurance, Part I

With logarithmic Bernoulli utility function, your utility with the insurance policy is

$$\ln(100000 - x),$$

and your expected utility without insurance is

$$0.95 \ln(100000) + 0.05 \ln(50000).$$

Equating these two values to find the value of x^* that leaves you indifferent between buying and not buying insurance yields

$$\ln(100000 - x^*) = 0.95 \ln(100000) + 0.05 \ln(50000).$$

Hence

$$100000 - x^* = \exp\left[0.95 \ln(100000) + 0.05 \ln(50000)\right]$$

or

$$x^* = 100000 - \exp\left[0.95 \ln(100000) + 0.05 \ln(50000)\right] = 3406.37,$$

implying that you would be willing to pay a premium of up to \$3406.37 on the insurance policy.

3. Insurance, Part II

With the addition 1 percent chance of the biggest disaster, x^* will have to satisfy

$$\ln(100000 - x^*) = 0.94 \ln(100000) + 0.05 \ln(50000) + 0.01 \ln(1).$$

Since $\ln(1) = 0$,

$$100000 - x^* = \exp\left[0.94 \ln(100000) + 0.05 \ln(50000)\right]$$

or

$$x^* = 100000 - \exp\left[0.94 \ln(100000) + 0.05 \ln(50000)\right] = 13910.83,$$

implying that you would be willing to pay a premium of up to \$13910.83 on the insurance policy.