Solutions to Problem Set 1

ECON 337901 - Financial Economics
Boston College, Department of Economics

Peter Ireland
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1. Profit Maximization

The first-order condition for the firm's problem,

$$\max_{n} \; n^{\alpha} - wn,$$

can be found by differentiating the objective function by the choice variable and setting the result equal to zero:

$$\alpha(n^*)^{\alpha-1} - w = 0.$$

Adding w to both sides of the first-order condition and then dividing both sides by α yields

$$(n^*)^{\alpha-1} = \frac{w}{\alpha}.$$

Finally, raising both sides of this last equation to the power $1/(\alpha - 1)$ leads to the solution

$$n^* = \left(\frac{w}{\alpha}\right)^{1/(\alpha-1)}.$$

Since $0 < \alpha < 1$, the exponent on the right-hand side is less than zero. This implies that when the wage rate w goes up, the number of workers hired n^* goes down. In fact, this equation for n^* describes the firm's demand curve for labor, showing the usual, inverse relationship between the wage and the number of workers hired.

2. Farming

The first-order condition for the farmer's problem,

$$\max_{h} \; \alpha \ln(h) - \beta h,$$

can be found by differentiating the objective function by the choice variable and setting the result equal to zero:

$$\frac{\alpha}{h^*} - \beta = 0.$$

Adding β to both sides of the first order condition, multiplying both sides by h^*, and then dividing both sides by β yields the solution

$$h^* = \frac{\alpha}{\beta}.$$

Since α and β are both positive, an increase in β implies that h^* goes down: the farmer's stronger distaste for working leads him or her to work less at the expense, of course, of consuming less as well.