

ECON 337901

FINANCIAL ECONOMICS

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2 Overview of Asset Pricing Theory

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Pricing Safe Cash Flows

A T -year discount bond is an asset that pays off \$1, for sure, T years from now.

If this bond sells for \$ P_T today, the annualized return from buying the bond today and holding it to maturity is

$$1 + r_T = \left(\frac{1}{P_T} \right)^{1/T}.$$

Hence, the bond price and the interest rate are related via

$$P_T = \frac{1}{(1 + r_T)^T}.$$

Pricing Safe Cash Flows

Since, for a T -period discount bond,

$$P_T = \frac{1}{(1 + r_T)^T},$$

the interest rate equates today's price of the bond to the present discounted value of the future payments made by the bond.

US Treasury bills, that is, US government bonds with maturities less than one year, are structured as discount bonds.

Pricing Safe Cash Flows

A T -year coupon bond is an asset that makes an annual interest (coupon) payment of $\$C$ each year, every year, for the next T years, and then pays off $\$F$ (face or par value), for sure, T years from now.

US Treasury notes and bonds, with maturities of more than one year, are structured as coupon bonds.

Pricing Safe Cash Flows

Notice that a coupon bond can be viewed as a bundle, or **portfolio** of discount bonds, since the cash flows from a T -year coupon bond can be replicated by buying

C one-year discount bonds

C two-year discount bonds

...

C T -year discount bonds

F more T -year discount bonds

Pricing Safe Cash Flows

And if both discount and coupon bonds are traded, then the price of the coupon bond must equal the price of the portfolio of discount bonds.

If the coupon bond was cheaper than the portfolio of discount bonds, one could sell the discount bonds, buy the coupon bond, and thereby profit.

If the coupon bond was more expensive than the portfolio of discount bonds, one could sell the coupon bond, buy the discount bonds, and thereby profit.

Pricing Safe Cash Flows

Building on this insight, the price P_T^C of the coupon bond must satisfy

$$\begin{aligned} P_T^C &= CP_1 + CP_2 + \dots + CP_T + FP_T \\ &= \frac{C}{1+r_1} + \frac{C}{(1+r_2)^2} + \dots + \frac{C}{(1+r_T)^T} + \frac{F}{(1+r_T)^T} \end{aligned}$$

Today's price of the coupon bond equals the present discounted value of the future payments made by the bond.

Pricing Safe Cash Flows

$$P_T^C = \frac{C}{1+r_1} + \frac{C}{(1+r_2)^2} + \dots + \frac{C}{(1+r_T)^T} + \frac{F}{(1+r_T)^T}$$

Note that the interest rates used to compute the present value are those on the **discount bonds**

The **yield to maturity** defined by the value r that satisfies

$$P_T^C = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \dots + \frac{C}{(1+r)^T} + \frac{F}{(1+r)^T}$$

is a measure of the interest rate on the coupon bond.

Pricing Safe Cash Flows

In fact, the US Treasury allows financial institutions to break US Treasury coupon bonds down into portfolios of separately-traded discount bonds.

These securities are called US Treasury STRIPS (Separate Trading of Registered Interest and Principal of Securities).

Pricing Safe Cash Flows

Next, consider an asset that generates an arbitrary stream of safe (riskless) cash flows C_1, C_2, \dots, C_T , over the next T years.

To simplify the task of “pricing” this asset, we might view it as a portfolio of more basic assets: one that pays C_1 for sure in one year, one that pays C_2 for sure in two years, \dots , and one that pays C_T for sure in T years.

The price of the multi-period asset must equal the sum of the prices of the more basic assets.

Pricing Safe Cash Flows

We've now reduced the problem of pricing any riskless asset to the simpler problem of pricing a more basic asset that pays C_t for sure t years from now.

But this more basic asset has the same payoff as C_t t -year discount bonds. Its price P_t^A today must equal

$$P_t^A = C_t P_t = \frac{C_t}{(1 + r_t)^t},$$

the present discounted value of its cash flow.

Pricing Risky Cash Flows

Now consider a risky asset, with cash flows $\tilde{C}_1, \tilde{C}_2, \dots, \tilde{C}_T$ over the next T years that are **random variables** with values that are unknown today.

Again, we might simplify the task of pricing this asset, by viewing it as a portfolio of more basic assets, each of which makes a random payment \tilde{C}_t after t years, then summing up the prices of all of these more basic assets.

Pricing Risky Cash Flows

But we still have to deal with the fact that the payoff \tilde{C}_t is risky.

And that is what the modern theory of asset pricing, on which this course is based, is really all about.

Pricing Risky Cash Flows

In probability theory, if a **random variable** \tilde{X} can take on n possible values, X_1, X_2, \dots, X_n , with probabilities $\pi_1, \pi_2, \dots, \pi_n$, then the **expected value** of \tilde{X} is

$$E(\tilde{X}) = \pi_1 X_1 + \pi_2 X_2 + \dots + \pi_n X_n.$$

Pricing Risky Cash Flows

One approach to asset pricing replaces the random payoff \tilde{C}_t with its expected value $E(\tilde{C}_t)$ and then “penalizes” the fact that the payoff is random by either discounting it at a higher rate

$$P_t^A = \frac{E(\tilde{C}_t)}{(1 + r_t + \psi_t)^t}$$

or by reducing its value more directly as

$$P_t^A = \frac{E(\tilde{C}_t) - \Psi_t}{(1 + r_t)^t}$$

Pricing Risky Cash Flows

$$P_t^A = \frac{E(\tilde{C}_t)}{(1 + r_t + \psi_t)^t}$$

$$P_t^A = \frac{E(\tilde{C}_t) - \Psi_t}{(1 + r_t)^t}$$

The **capital asset pricing model** (CAPM), the **consumption capital asset pricing model** (CCAPM), and the **arbitrage pricing theory** (APT) will give us ways of determining values for the **risk premium** ψ_t or Ψ_t .

Pricing Risky Cash Flows

Another possibility is to break down the random payoff \tilde{C}_t into separate components $C_{t,1}, C_{t,2}, \dots, C_{t,n}$ delivered in n different “states of the world” that can prevail t years from now.

The risky asset that delivers the random payoff \tilde{C}_t t years from now can itself be viewed as a portfolio of contingent claims: $C_{t,1}$ contingent claims for state 1, $C_{t,2}$ contingent claims for state 2, \dots , and $C_{t,n}$ contingent claims for state n .

Pricing Risky Cash Flows

This **Arrow-Debreu** approach to asset pricing then computes

$$P_t^A = q_{t,1}C_{t,1} + q_{t,2}C_{t,2} + \dots + q_{t,n}C_{t,n}$$

where $q_{t,i}$ is the price today of a contingent claim that delivers one dollar if state i occurs t years from now and zero otherwise.

This approach uses contingent claims as the “basic building blocks” for risky assets, in the same way that discount bonds can be viewed as the building blocks for coupon bonds.

Pricing Risky Cash Flows

Yet another possibility is to “distort” the probabilities so as to down-weight favorable outcomes and over-weight adverse outcomes, to use these distorted probabilities to re-calculate

$$\hat{E}(C_t) = \hat{\pi}_1 C_{t,1} + \hat{\pi}_2 C_{t,2} + \dots + \hat{\pi}_n C_{t,n},$$

and then to price the asset based on the distorted expectation:

$$P_t^A = \frac{\hat{E}(C_t)}{(1 + r_t)^t}.$$

The **martingale** approach to asset pricing will tell us how to do this.

Two Perspectives on Asset Pricing

Although all are designed to accomplish the same basic goal – to value risky cash flows – these different theories of asset pricing can be grouped under two broad headings.

No-arbitrage theories take the prices of some assets as given and use those to determine the prices of other assets.

Equilibrium theories price all assets based on the principles of microeconomic theory.

Two Perspectives on Asset Pricing

No-arbitrage theories require fewer assumptions and are sometimes easier to use.

We've already used no-arbitrage arguments, for example, to price stocks and bonds as portfolios of contingent claims and to price coupon bonds as portfolios of discount bonds.

Two Perspectives on Asset Pricing

But no-arbitrage theories raise questions that only equilibrium theories can answer.

Where do the prices of the basic securities come from?

And how do asset prices relate to economic fundamentals?

Two Perspectives on Asset Pricing

	Equilibrium	No-Arbitrage
Risk Premia	CAPM, CCAPM	APT
Contingent Claims	A-D	A-D
Distorted Probabilities		Martingale

Two Perspectives on Asset Pricing

A Introduction

- 1 Mathematical and Economic Foundations
- 2 Overview of Asset Pricing Theory

B Decision-Making Under Uncertainty

- 3 Making Choices in Risky Situations
- 4 Measuring Risk and Risk Aversion

C The Demand for Financial Assets

- 5 Risk Aversion and Investment Decisions
- 6 Modern Portfolio Theory

Two Perspectives on Asset Pricing

D Classic Asset Pricing Models

7 The Capital Asset Pricing Model

8 Arbitrage Pricing Theory

E Arrow-Debreu Pricing

9 Arrow-Debreu Pricing: Equilibrium

10 Arrow-Debreu Pricing: No-Arbitrage

F Extensions

11 Martingale Pricing

12 The Consumption Capital Asset Pricing Model