Midterm Exam

ECON 337901 - Financial Economics | Peter Ireland
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Tuesday, March 19, 10:30 - 11:45am

This exam has five questions on four pages; before you begin, please check to make sure that your copy has all five questions and all four pages. Please note, as well, that each of the five questions has three parts. The five questions will be weighted equally in determining your overall exam score.

Please circle your final answer to each part of each question after you write it down, so that I can find it more easily. If you show the steps that led you to your results, however, I can award partial credit for the correct approach even if your final answers are slightly off.

1. Profit Maximization

Consider a firm that hires n workers in order to produce y units of output according to the production function

$$y = n^{1/2}.$$

Let w denote the wage that the firm must pay each worker in a competitive labor market. Then the firm chooses n to maximize profits, defined as usual to equal revenues minus costs:

$$\max_n n^{1/2} - wn.$$

This is an unconstrained optimization problem, with an objective function $F(n) = n^{1/2} - wn$ that is concave, so that the first-order condition is both necessary and sufficient for the value of n^* that maximizes profits.

a. Write down the first-order condition for n^*, using the rules of differentiation to find $F'(n^*)$ for this example.

b. Next, rearrange the first-order condition to get an equation that shows how the firm's optimal choice n^* depends on the wage rate w.

c. Finally, use your solution from part (b) to answer the question: when w goes up, does n^* rise or fall?

2. Consumer Optimization Under Uncertainty

Consider a consumer making choices under uncertainty in an environment with two periods, today ($t = 0$) and next year ($t = 1$), and two states, good and bad, in period $t = 1$. Let c_0 denote consumption today and c_1^G and c_1^B denote consumption in the good and bad states next year. The consumer's expected utility is then

$$u(c_0) + \beta\pi u(c_1^G) + \beta(1-\pi)u(c_1^B),$$

where β is a discount factor that captures the consumer's degree of patience or impatience and π denotes the probability that the good state occurs next year. Similarly, let Y_0, Y_1^G, and Y_1^B denote the consumer's income today and in the good and bad states next year. The consumer's budget constraint is then

$$Y_0 + q^G Y_1^G + q^B Y_1^B \geq c_0 + q^G c_1^G + q^B c_1^B,$$

where q^G is the price of a contingent claim for the good state next year and q^B is the price of a contingent claim for the bad state next year.

Now specialize this problem by setting $\beta = 1/2$, $\pi = 1/2$, $1 - \pi = 1/2$, $Y_0 = 60$, $Y_1^G = 80$, $Y_1^B = 40$, $q^G = 1/4$, and $q^B = 1/4$. Assume, as well, that the utility function $u(c) = \ln(c)$ takes the natural log form, so that $u'(c) = 1/c$. With these settings, the consumer's problem becomes, more specifically, one of choosing c_0, c_1^G, and c_1^B to maximize expected utility

$$\ln(c_0) + \left(\frac{1}{4}\right)\ln(c_1^G) + \left(\frac{1}{4}\right)\ln(c_1^B)$$

subject to the budget constraint

$$90 \geq c_0 + \left(\frac{1}{4}\right)c_1^G + \left(\frac{1}{4}\right)c_1^B.$$

a. Write down the Lagrangian for the consumer's problem.

b. Next, write down the first-order conditions for the consumer's optimal choices c_0^*, c_1^{G*}, and c_1^{B*} of c_0, c_1^G, and c_1^B.

c. Finally, use your first-order conditions from part (b) above, together with the binding budget constraint

$$90 = c_0^* + \left(\frac{1}{4}\right)c_1^{G*} + \left(\frac{1}{4}\right)c_1^{B*},$$

to find the numerical values of c_0^*, c_1^{G*}, and c_1^{B*}.

3. Stocks, Bonds, and Contingent Claims

Consider an economy in which two assets are initially traded. A stock sells for $q^s = 2$ today (at $t = 0$) and pays off a large dividend $d^G = 4$ in a good state next year ($t = 1$) and a smaller dividend $d^B = 2$ in a bad state next year. A bond sells for $q^b = 0.75$ today and pays off one dollar for sure, in both the good and the bad states next year.

a. What would be the price q^G today (at $t = 0$) of a contingent claim for the good state, which pays off one dollar in the good state next year (at $t = 1$) and zero in the bad state next year, if there are no arbitrage opportunities across the markets for the stock, bond, and contingent claim?

b. What would be the price q^B today (at $t = 0$) of a contingent claim for the bad state, which pays off one dollar in the bad state next year (at $t = 1$) and zero in the good state next year, if there are no arbitrage opportunities across the markets for the stock, bond, and contingent claim?

c. Suppose that a new asset begins trading, which pays off $X^G = 3$ in the good state and $X^B = 1$ in the bad state next year (at $t = 1$). What would be the price q^A of this asset today (at $t = 0$), if there are no arbitrage opportunities across all markets for the stock, bond, contingent claims, and this new asset?

4. Option Pricing

Consider another economy in which two assets are initially traded – note that the numbers here are slightly different from those in question 3. Here, a stock sells for $q^s = 2$ today (at $t = 0$), and sells for a high price $P^G = 5$ in a good state next year ($t = 1$) and a smaller price $P^B = 3$ in a bad state next year. A bond sells for $q^b = 0.60$ today and pays off one dollar for sure, in both the good and the bad states next year.

a. Now consider a call option, which gives the holder the right, but not the obligation, to purchase the stock at the strike price $K = 2$ next year (at $t = 1$). What are the payoffs from the option in the good state and bad state next year?

b. How many shares of stock s and bonds b would an investor have to buy or sell short to replicate the payoffs on this call option?

c. What would be the price q^o of the option today (at $t = 0$), if there are no arbitrage opportunities across the markets for the stock, bond, and option?

5. Pricing Risk-Free Assets

Suppose that a one-year discount bond that pays off one dollar for sure one year from now sells for $P_1 = 0.90$ today, a two-year discount bond that pays off one dollar for sure two years from now sells for $P_2 = 0.80$ today, and a three-year discount bond that pays off one dollar for sure three years from now sells for $P_3 = 0.70$ today.

a. Consider a three-year coupon bond that makes an annual interest (coupon) payment of 100 dollars at the end of each of the next three years and also returns an additional amount (face value) of 1000 dollars at the end of the third year. What will the price of this bond be if there are no arbitrage opportunities across the markets for discount and coupon bonds?

b. Next, consider a risk-free asset that pays the holder 100 dollars for sure two years from now and 100 dollars for sure three years from now. What will the price of this asset be if there are no arbitrage opportunities across markets for all risk-free assets?

c. Finally, consider another risk-free asset that pays the holder 100 dollars for sure two years from now but then requires the holder *to pay* 100 dollars for sure three years from now. What will the price of this asset be if there are no arbitrage opportunities across markets for all risk-free assets?