Final Exam

ECON 337901 - Financial Economics — Peter Ireland
Boston College, Department of Economics — Spring 2020

Due Thursday, May 7, 12noon

This exam has five questions on five pages, including this cover sheet. Before you begin, please check to make sure that your copy has all five questions and all five pages. Please note, as well, that each of the five questions has three parts. The five questions will be weighted equally in determining your overall exam score.

Please circle your final answer to each part of each question after you write it down, so that I can find it more easily. If you show the steps that led you to your results, however, I can award partial credit for the correct approach even if your final answers are slightly off.

This is an open-book exam, meaning that it is fine for you to consult your notes, the class videos, and all material from the course webpage when working on your answers to the questions. I expect you to work independently on the exam, however, without discussing the questions or answers with anyone else, in person or electronically, inside or outside of the class; the answers you submit must be yours and yours alone.

1. Criteria for Choice Over Risky Prospects

The table below shows the percentage returns on two risky assets, asset 1 and asset 2, in an economic environment in which there are two future states: a good state that occurs with probability $\pi = 1/2$ and a bad state that occurs with probability $1 - \pi = 1/2$.

	Percentage Return in	
Asset	Good State	Bad State
Asset 1	15	5
Asset 2	12	6

a. Does either asset display state-by-state dominance over the other? If so, which one?

b. Does either asset display mean-variance dominance over the other? If so, which one?

c. Assuming the risk-free interest rate is zero, does either asset have a Sharpe ratio that is larger than the other? If so, which one?

2. Insurance

Suppose that a consumer with initial income of \$100,000 faces a 1 percent ($\pi_1 = 0.01$) probability of experiencing a loss that is equal in value to 90 percent of his or her income. Suppose also that this consumer's preferences are described by a von Neumann-Morgenstern expected utility function with Bernoulli utility function of the natural log form:

$$u(Y) = \ln(Y).$$

a. Without insurance, the consumer will have income of \$100,000 in a "good state" that occurs with probability $1 - \pi_1 = 0.99$ and income of only \$10,000 in a "bad state" that occurs with probability $\pi_1 = 0.01$. Write down an expression for the consumer's expected utility in this case without insurance.

b. If the consumer is able to buy insurance again the possibility of the loss, he or she will have income of \$100,000-x for sure, where x is the cost of buying the insurance. Write down an expression for the consumer's expected utility in this case with insurance.

c. Now combine your answers from parts (a) and (b) above to answer the question: what is the numerical value of the largest amount x^* that the consumer will be willing to pay for the insurance policy?

3. Wealth, Risk Aversion, and Portfolio Allocation

Suppose that an investor has initial wealth $Y_0 = 1000$. The investor allocates the amount a to stocks, which provide return $r_G = 0.25$ (a 25 percent gain) in a good state next year that occurs with probability $\pi = 1/2$ and return $r_B = -0.20$ (a 20 percent loss) in a bad state next year that occurs with probability $1 - \pi = 1/2$.

The investor allocates the remaining amount $Y_0 - a$ to a risk-free bond. Suppose, partly for simplicity but also because interest rates on short-term US government bonds are very close to zero these days, that the bond in this example provides a return $r_f = 0$ in both states next year. This means that every dollar that the investor allocates to the bond provides a dollar back for sure next year; the money is safe but does not earn any interest. Assume, finally, that the investor's preferences can be described by a von Neumann-Morgenstern expected utility function, with Bernoulli utility function of the constant relative risk aversion form

$$u(Y) = \frac{Y^{1-\gamma} - 1}{1 - \gamma},$$

where, in particular, $\gamma = 1/2$.

Then, in general, the investor's problem can be stated mathematically as

$$\max_a \ \pi \left\{ \frac{[(1 + r_f)Y_0 + a(r_G - r_f)]^{1-\gamma} - 1}{1 - \gamma} \right\} + (1 - \pi) \left\{ \frac{[(1 + r_f)Y_0 + a(r_B - r_f)]^{1-\gamma} - 1}{1 - \gamma} \right\},$$

but with the specific values of $Y_0 = 1000$, $r_G = 0.25$, $r_B = -0.20$, $r_f = 0$, $\pi = 1/2$, $1 - \pi = 1/2$, and $\gamma = 1/2$ given above, the problem can be written more simply as

$$\max_a \ \frac{1}{2} \left[\frac{(1000 + 0.25a)^{1/2} - 1}{1/2} \right] + \frac{1}{2} \left[\frac{(1000 - 0.20a)^{1/2} - 1}{1/2} \right].$$

a. Write down the first-order condition for the investor's optimal choice of a^*, the amount allocated to stocks. Then, use the first-order condition to find the numerical value of a^*.

b. Suppose that, instead of having initial wealth $Y_0 = 1000$, the investor has initial wealth $Y_0 = 2000$. What is the numerical value of a^* now? *Note:* Although you can find the new value of a^* by solving the investor's problem all over again with $Y_0 = 2000$ in place of $Y_0 = 1000$, you might also just recall that we discussed in class what happens to the value of a^* that solves this problem when initial wealth doubles, given that the Bernoulli utility function takes the constant relative risk aversion form.

c. Go back to assuming that $Y_0 = 1000$, but suppose now that instead of $\gamma = 1/2$, the investor's constant coefficient of relative risk aversion is $\gamma = 2$. Will the value of a^* that solves the investor's problem in this case be greater than, less than, or equal to the value of a^* that you calculated in answering part (a)? *Note:* For this part (c), you don't have to actually calculate the new value of a^*, you just have to say how it compares to the value from part (a).

4. A Risk-Return Tradeoff

Suppose that an investor allocates the share w of his or her initial wealth to a stock mutual fund with risky (random) return $\tilde{r}$, expected return μ_r, and standard deviation of its random return σ_r, and the remaining share $1-w$ to a risk-free bond with known (non-random) return r_f. In answering the questions below, assume that $\mu_r > r_f$ and that $\sigma_r > 0$, so that stocks, though risky, have a higher expected return than bonds. Assume, as well, that w satisfies $w \geq 0$, so that the share of wealth allocated to stocks can't be negative; this simplifying assumption rules out more complicated cases where the investor decides to sell stocks short.

a. Write down a formula for the expected return μ_p on the investor's overall portfolio and use this formula to answer the question: if the investor chooses a larger value of w, thereby allocating more to stocks, will μ_p rise, fall, or stay the same?

b. Next, write down a formula for the standard deviation σ_p of the random return on the investor's overall portfolio and use this formula to answer the question: if the investor chooses a larger value of w, thereby allocating more to stocks, will σ_p rise, fall, or stay the same?

c. Finally, using your answers to parts (a) and (b), write down the formula that relates μ_p directly to σ_p, and thereby summarizes the risk-return tradeoff faced by the investor, without making reference to w. *Hint:* Recall from our class discussions that, when plotted in a graph with σ_p on the x-axis and μ_p on the y-axis, this relationship should be linear with y-intercept equal to r_f and slope equal to the risky mutual fund's Sharpe ratio.

5. Using the CAPM to Price a Risky Cashflow

Suppose that the risk-free interest rate on one-year government bonds is $r_f = 0.01$ (one percent) and the risky return $\tilde{r}_M$ over the next year on the stock market as a whole has expected value $E(\tilde{r}_M) = 0.07$ (seven percent) and variance $\sigma_M^2 = 0.03$.

Recall from our discussions in class that if an asset, call it "asset A," sells for P^A today and makes a random payoff $\tilde{C}^A$ one year from now, its random return is equal to

$$\tilde{r}_A = \frac{\tilde{C}^A}{P^A} - 1$$

and its expected return is equal to

$$E(\tilde{r}_A) = \frac{E(\tilde{C}^A)}{P^A} - 1.$$

Suppose that asset A's random return $\tilde{r}_A$ is normally distributed, with variance $\sigma_A^2 = 0.04$ and covariance $\mathrm{cov}(\tilde{r}_A, \tilde{r}_M) = \sigma_{AM} = 0.02$ with the market's return. Finally, suppose that asset A's expected payoff is $E(\tilde{C}^A) = 105$.

a. Using the data given above, compute the numerical value of asset A's capital asset pricing model (CAPM) beta β_A.

b. Using your answer to part (a), compute the numerical value of the price P^A at which, according to the CAPM, asset A should sell for today.

c. Suppose, instead, that asset A's random return has covariance $\mathrm{cov}(\tilde{r}_A, \tilde{r}_M) = \sigma_{AM} = 0.03$ with the market's return. If all the other data stay the same as above, what does the CAPM predict: will today's price P^A of asset A be higher than, the same as, or lower than the answer you calculated in part (b)? *Note:* For this part (c), you don't have to actually calculate the new value of P^A, you just have to say how it compares to the value from part (b).